Tensor Networks for Advanced Filtering and Decision-Making: A Quantum-Inspired Framework

Tensor networks have emerged as a transformative mathematical framework that bridges quantum mechanics and classical machine learning, offering unprecedented capabilities for advanced filtering and decision-making protocols. By leveraging CNOT and SWAP gates to create quantum entanglement between variable pairs, these systems can capture complex correlations and enable serendipitous knowledge discovery in ways that classical methods cannot achieve. [1] [2]

Foundation: Tensor Networks as Quantum-Inspired Tools

Tensor networks, originally developed to simulate quantum many-body systems, efficiently represent high-dimensional data by exploiting low-rank structure. Matrix Product States (MPS) and more complex architectures like Tree Tensor Networks (TTN) and Projected Entangled Pair States (PEPS) reduce parameter complexity from exponential to polynomial, enabling practical implementation on classical hardware while retaining quantum-inspired advantages. [3] [4] [1]

The key innovation lies in representing probability distributions as quantum states, where the squared amplitudes correspond to probabilities. This quantum probabilistic interpretation allows tensor networks to capture correlations between variables through entanglement structure—a property fundamentally different from classical correlation measures. [5] [1]

CNOT and SWAP Gates: Creating Entanglement Between Variables

CNOT Gate Implementation

The Controlled-NOT (CNOT) gate serves as the fundamental entangling operation in quantum circuits. When applied to two qubits representing different variables, it creates quantum correlations by flipping the target qubit if and only if the control qubit is in state $|1\rangle$ [6] [7]. In tensor network formulations:

$$\text{CNOT} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

For filtering applications, CNOT gates enable conditional feature selection where the relevance of one variable depends on the state of another. This creates information dependencies that classical methods struggle to capture efficiently. [8] [9] [10]

SWAP Gate Operations

SWAP gates exchange quantum states between qubits, implemented through three consecutive CNOT operations: CNOT(q_1,q_2), CNOT(q_2,q_1), CNOT(q_1,q_2). In decision-making protocols, SWAP gates allow dynamic rearrangement of variable ordering to optimize circuit depth and explore different relationship structures.[6] [4] [11] [8]

The entanglement created by these gates manifests in the tensor network as bond dimensions connecting different parts of the network. Higher bond dimensions indicate stronger correlations and more complex entanglement structures.[12] [5]

Advanced Filtering with Entanglement-Based Feature Extraction

Entanglement Entropy as Feature Importance

Recent breakthroughs demonstrate that entanglement entropy in MPS-based classifiers directly quantifies feature importance. When an MPS is trained on data, the Single-site Entanglement Entropy (SEE) and Bipartite Entanglement Entropy (BEE) reveal which features contribute most to classification:[13] [5]

- **High SEE regions** correspond to critical features that encode the main distinguishing information
- **Low SEE regions** indicate redundant or uninformative features that can be filtered out

This provides a quantum-inspired alternative to classical sensitivity analysis, enabling automatic feature extraction based on the learned entanglement structure.[14] [5]

Tensor Network Kalman Filtering

For dynamic filtering applications, tensor network Kalman filters leverage low-rank tensor decompositions to handle high-dimensional state spaces. The recently developed tensor network square root Kalman filter solves the critical issue of filter divergence while estimating up to 4^{14} parameters on standard hardware. This enables real-time filtering in systems where classical methods fail due to dimensionality constraints.[15] [16]

The filtering operation decomposes the covariance matrix using tensor train format, maintaining numerical stability through:

$$P_k = L_k L_k^T$$

where L_k is represented as a tensor train with controlled rank.[15]

Decision-Making Protocols with Hybrid Quantum-Classical Architecture

Variational Quantum Circuits for Decision Optimization

Hybrid quantum-classical decision protocols combine variational quantum circuits (VQCs) with classical optimizers to create flexible decision-making systems. The architecture consists of: [17] [18] [19]

1. **Quantum Feature Encoding**: Classical data is mapped to quantum states using feature maps that exploit CNOT gates to encode correlations between input features[20]
2. **Parameterized Quantum Circuits**: Layers of single-qubit rotations interleaved with CNOT entangling gates create expressive quantum states:
$$|\psi(\theta)\rangle = U(\theta)|0\rangle^{\otimes n}$$
3. **Classical Measurement and Optimization**: Quantum measurements collapse to classical outcomes, which are fed into classical optimization algorithms to update circuit parameters[19] [17]

This hybrid approach is essential because pure quantum systems cannot perform decision-making without classical components—the no-cloning theorem prevents copying quantum states for comparison, a fundamental requirement for deliberation. [21] [22]

Actor-Critic with Tensor Networks (ACTeN)

The ACTeN framework integrates tensor networks with reinforcement learning for dynamical optimization. The policy and value functions are parameterized as tensor networks, enabling: [23]

- Efficient representation of factorizable state-action spaces
- Polynomial scaling instead of exponential growth
- Application to complex problems like sampling rare trajectories in stochastic systems[23]

This demonstrates how tensor networks enable decision-making in scenarios with exponentially large state spaces, previously intractable for classical methods.

Serendipitous Discovery: Quantum-Inspired Knowledge Exploration

Knowledge Graph Completion via Tensor Factorization

Tensor decomposition methods enable serendipitous knowledge discovery by predicting missing relationships in knowledge graphs. Neural Tensor Networks (NTN) use a bilinear tensor product to score relationship triplets: [24] [25] [26] [27]

$$g(e_1, R, e_2) = u_R^T f(e_1^T W_R^{[1:k]} e_2 + V_R, 1e_1 + V_R, 2e_2 + b_R)$$

where $W_R^{[1:k]}$ is a third-order tensor that captures complex interactions between entities. This enables: [25] [24]

- **Transitive reasoning**: Discovering indirect connections across multiple relationships
- **Entity generalization**: Sharing statistical strength between similar entities through word vector initialization

- **Novel fact prediction**: Completing knowledge bases by inferring likely but unrecorded relationships[27] [25]

Tucker decomposition-based methods like TuckER achieve state-of-the-art performance (86-90% accuracy) in predicting missing knowledge graph entries, enabling discovery of unexpected connections that human experts might overlook.[28] [29]

Quantum Walk Search for Serendipitous Exploration

Quantum walk algorithms provide quadratic speedup over classical random walks for graph search, enabling more efficient exploration of knowledge spaces. The continuous-time quantum walk Hamiltonian:[30] [31] [32]

$$H = -\gamma L - |w\rangle\langle w|$$

where L is the graph Laplacian, allows simultaneous exploration of multiple paths through superposition. For serendipitous discovery:[32] [33]

- **Multi-path exploration**: Quantum walks explore multiple knowledge paths simultaneously, increasing the probability of finding unexpected but relevant connections[34] [30]
- **Ordered marked nodes**: Recent extensions handle scenarios where discoveries have temporal or causal ordering, crucial for understanding research dependencies[32]
- **Dynamic labeling**: The algorithm can track different categories of discoveries, enabling structured exploration of complex knowledge domains[32]

LLM-Assisted Serendipity Assessment

Recent frameworks evaluate Large Language Models' ability to facilitate serendipitous knowledge discovery through three phases:[35]

1. **Knowledge Retrieval**: Accessing relevant information from knowledge bases
2. **Reasoning**: Making logical connections between disparate facts
3. **Exploratory Search**: Identifying surprising associations that lead to novel insights

While current models excel at retrieval, they struggle significantly with serendipity exploration, highlighting the need for hybrid systems that combine neural networks with structured quantum-inspired search algorithms.[36] [35]

Practical Implementation Framework

Software Architecture

Modern tensor network frameworks provide accessible implementation tools:[37] [38] [39] [40]

- **ITensor (C++/Julia)**: Mature library with tensor diagram-based interface, ideal for physics-inspired applications[37]
- **TensorCircuit (Python)**: Built on JAX/TensorFlow/PyTorch, supports automatic differentiation and JIT compilation for quantum circuit simulation[39]

- **Cytnx (C++/Python)**: Unified interface with GPU support via cuQuantum for large-scale tensor network calculations [38]

These frameworks enable researchers to implement MPS-based machine learning, quantum circuit simulation, and tensor network optimization without deep expertise in quantum mechanics. [41] [40]

Algorithm Workflow

A complete tensor network filtering and decision-making pipeline includes:

1. Data Encoding Phase:

```
Input data → Feature map → Quantum state |ψ(x)⟩
```

Features are encoded using amplitude encoding (logarithmic qubit requirement) or basis encoding, with CNOT gates creating initial entanglement structure. [42] [43]

2. Entanglement Generation:

```
Apply variational layers: R_y(θ₁) ⊗ R_x(θ₂) → CNOT → R_y(θ₃) ⊗ R_x(θ₄) → CNOT
```

Parameterized rotations followed by entangling CNOT gates build up complex correlation patterns. [44] [43]

3. Measurement and Filtering:

```
Measure Pauli operators → Estimate entanglement entropy → Rank features by SEE
```

Local measurements provide information about entanglement structure without full state tomography. [45] [5]

4. Decision Output:

```
Tensor network contraction → Classical post-processing → Decision/prediction
```

Efficient contraction algorithms compute expectation values in polynomial time. [4] [11] [46]

Hybrid Quantum-Classical Integration

Recent research demonstrates that effective decision-making requires both quantum and classical resources: [47] [22] [21]

- **Quantum advantages**: Superposition enables parallel exploration of solution spaces; entanglement captures correlations impossible to represent classically
- **Classical necessities**: Stable information storage, copying for comparison, and measurement to collapse quantum states into definite outcomes

Optimal implementations use classical controllers to prepare quantum states, execute quantum operations, then process measurement results classically. This "middle ground" appears fundamental rather than a temporary limitation.[48] [21]

Applications and Performance

Feature Selection and Dimensionality Reduction

Quantum-inspired feature selection using quantum annealing provides one-shot solutions to combinatorial optimization problems. D-Wave's hybrid quantum computing service achieves:[49] [50] [51] [52]

- **500× speedup** compared to recursive feature elimination on large datasets[53]
- **Global optimization** avoiding greedy local decisions of classical methods[51]
- **Automatic rank determination** in tensor decompositions without manual tuning[54]

Drug Discovery and Molecular Design

Quantum machine learning accelerates drug discovery through:[43] [55] [56] [57]

- **Molecular property prediction**: VQCs predict binding affinity with fewer parameters than classical neural networks
- **Generative design**: Hybrid quantum-classical VAEs generate novel peptide sequences with optimized properties[43]
- **Quantum chemistry simulation**: Tensor network representations enable accurate modeling of molecular interactions at polynomial cost[55] [58]

Real-Time Optimization

Quantum-inspired algorithms solve real-world optimization problems efficiently:[59] [53]

- **Delivery routing**: DENSO Mk-D solves 5-million-variable problems in 6 minutes, 500× faster than classical methods[53]
- **Adaptive filtering**: Third-order tensor decomposition RLS algorithms provide low-complexity solutions for system identification[60] [61]
- **Motion planning**: Tensor-train value iteration handles high-dimensional stochastic control with polynomial complexity[62]

Challenges and Future Directions

Current Limitations

1. **Quantum hardware constraints**: Near-term quantum devices have limited qubit counts and high error rates, restricting practical implementations[55] [21]
2. **Classical simulation costs**: While tensor networks reduce complexity from exponential to polynomial, simulating large entangled systems remains computationally intensive[11] [4]

3. **Bond dimension scaling**: Highly entangled states require large bond dimensions, potentially negating computational advantages[63] [46]
4. **Serendipity evaluation**: Quantifying and optimizing for unexpected discovery remains an open challenge, lacking standardized metrics[35] [36]

Emerging Solutions

Recent advances address these limitations:

- **Dynamic circuits**: Mid-circuit measurement with feed-forward enables efficient long-range entanglement creation, outperforming unitary-only approaches on 100+ qubit systems[9] [10]
- **Adaptive tensor networks**: Rank-adaptive algorithms automatically adjust bond dimensions based on required accuracy, balancing precision and efficiency[64] [62]
- **Hybrid error mitigation**: Combining tensor network methods with error correction codes improves reliability on noisy quantum hardware[65] [66]
- **LLM-augmented exploration**: Integrating large language models with tensor network search enables more effective serendipitous discovery through natural language reasoning[67] [35]

Synthesis: Integrated Architecture

An optimal advanced filtering and decision-making system combines:

Layer 1: Quantum-Inspired Encoding

- Feature maps with CNOT/SWAP gates create entangled representations
- Tensor train format maintains polynomial complexity
- Entanglement entropy identifies important features

Layer 2: Hybrid Processing

- Variational quantum circuits explore solution spaces via superposition
- Classical controllers manage quantum resources and process measurements
- Tensor network contractions compute expectations efficiently

Layer 3: Serendipitous Discovery

- Quantum walk algorithms enable parallel exploration of knowledge graphs
- Tensor factorization predicts missing relationships
- LLM reasoning identifies surprising but relevant connections

Layer 4: Decision Output

- Actor-critic reinforcement learning optimizes long-term objectives
- Hybrid quantum-classical architecture ensures reliable decision-making
- Continuous learning adapts to new patterns and discoveries

This integrated approach makes quick and deep research more accessible by:

- **Reducing computational requirements** through tensor compression (up to 10-million-fold speedup)[68] [69]
- **Enabling automatic discovery** of complex patterns without manual feature engineering[5] [67]
- **Providing interpretability** through entanglement analysis, unlike black-box neural networks[70] [1]
- **Scaling to large systems** via distributed tensor network computations[11] [38]

The fusion of quantum entanglement principles with classical tensor representations creates a powerful new paradigm for intelligent information processing, bridging the gap between theoretical quantum advantages and practical classical implementation. As quantum hardware continues improving and tensor network algorithms become more sophisticated, these methods promise to unlock previously intractable problems in scientific discovery, optimization, and knowledge synthesis.[71] [65]

⁂

1. https://spj.science.org/doi/10.34133/icomputing.0061
2. https://papers.nips.cc/paper/6211-supervised-learning-with-tensor-networks
3. https://en.wikipedia.org/wiki/Tensor_network
4. https://quantum-journal.org/papers/q-2023-03-30-964/
5. https://www.frontiersin.org/journals/applied-mathematics-and-statistics/articles/10.3389/fams.2021.716044/full
6. https://fiveable.me/quantum-computing/unit-5/multi-qubit-gates-cnot-swap/study-guide/tWfv9yYrLkxvBrct
7. https://en.wikipedia.org/wiki/Controlled_NOT_gate
8. https://arxiv.org/pdf/2009.13247.pdf
9. https://arxiv.org/html/2308.13065v2
10. https://link.aps.org/doi/10.1103/PRXQuantum.5.030339
11. https://arxiv.org/abs/2310.03978
12. https://www.nature.com/articles/s41534-020-0281-5
13. https://cpl.iphy.ac.cn/article/10.1088/0256-307X/39/10/100701
14. https://tcmp2.ucas.ac.cn/index.php/en/publication/2021/252-entanglement-based-feature-extraction-by-tensor-network-machine-learning
15. https://arxiv.org/abs/2409.03276
16. https://www.sciencedirect.com/science/article/pii/S0005109825005916?dgcid=rss_sd_all
17. https://link.aps.org/doi/10.1103/PhysRevResearch.6.023110
18. https://www.quandela.com/resources/quantum-computing-glossary/variational-quantum-algorithm/
19. https://www.quera.com/glossary/variational-quantum-algorithm
20. https://apxml.com/courses/fundamentals-quantum-machine-learning/chapter-2-advanced-quantum-data-encoding/feature-map-expressibility-entanglement

21. https://thequantuminsider.com/2025/10/25/new-research-shows-decision-making-needs-both-quantum-and-classical-worlds/
22. https://cpb.iphy.ac.cn/EN/10.1088/1674-1056/adefd7
23. https://link.aps.org/doi/10.1103/PhysRevLett.132.197301
24. https://proceedings.neurips.cc/paper/2013/file/b337e84de8752b27eda3a12363109e80-Paper.pdf
25. https://nlp.stanford.edu/pubs/SocherChenManningNg_NIPS2013.pdf
26. https://arxiv.org/abs/1301.3618
27. https://papers.nips.cc/paper/5028-reasoning-with-neural-tensor-networks-for-knowledge-base-completion
28. https://aclanthology.org/D19-1522/
29. https://jmlr.org/papers/v18/16-563.html
30. https://en.wikipedia.org/wiki/Quantum_walk_search
31. https://link.aps.org/doi/10.1103/PhysRevLett.128.050501
32. https://www.nature.com/articles/s41598-024-51709-0
33. https://www.cs.umd.edu/~amchilds/talks/unm07.pdf
34. https://arxiv.org/abs/2510.04973
35. https://arxiv.org/html/2511.12472v1
36. https://research.rug.nl/files/108096977/a15_khalili.pdf
37. https://scipost.org/preprints/scipost_202007_00059v2/
38. https://arxiv.org/abs/2401.01921
39. https://github.com/tencent-quantum-lab/tensorcircuit
40. https://tensornetwork.org/software/
41. https://www.frontiersin.org/journals/applied-mathematics-and-statistics/articles/10.3389/fams.2022.806549/full
42. https://arxiv.org/abs/2406.17441
43. https://www.science.org/doi/10.1126/sciadv.adq8492
44. https://arxiv.org/abs/2508.04098
45. https://www.nature.com/articles/s41534-025-00970-w
46. https://link.aps.org/doi/10.1103/PhysRevLett.131.180601
47. https://arxiv.org/abs/2404.10389
48. https://www.nature.com/articles/s41598-025-13417-1
49. https://aws.amazon.com/marketplace/pp/prodview-bsrc3yuwgjbo4
50. https://www.spiedigitallibrary.org/conference-proceedings-of-spie/13028/130280C/Quantum-machine-learning-for-feature-selection-in-Internet-of-Things/10.1117/12.3013539.full
51. https://www.dwavequantum.com/media/bohmrx4p/feature-selection.pdf
52. https://arxiv.org/abs/2203.13261
53. https://www.denso.com/global/en/driven-base/tech-design/mk-d/
54. https://arxiv.org/html/2505.06203v3
55. https://www.sciencedirect.com/science/article/abs/pii/S135964462500176X

56. https://pubs.acs.org/doi/10.1021/acs.chemrev.4c00678
57. https://pmc.ncbi.nlm.nih.gov/articles/PMC8254374/
58. https://arxiv.org/abs/2409.15645
59. https://www.global.toshiba/ww/company/digitalsolution/articles/tsoul/tech/t0501.html
60. https://www.sciencedirect.com/science/article/abs/pii/S0165168423002906
61. https://ieeexplore.ieee.org/document/10694445/
62. https://www.roboticsproceedings.org/rss11/p15.pdf
63. https://pennylane.ai/qml/demos/tutorial_How_to_simulate_quantum_circuits_with_tensor_networks
64. https://arxiv.org/html/2506.19339v1
65. https://thequantuminsider.com/2025/08/05/global-tech-leaders-publish-review-on-tensor-networks-in-quantum-computing/
66. https://journals.aps.org/prresearch/abstract/10.1103/PhysRevResearch.7.013237
67. https://proceedings.mlr.press/v235/zeng24b.html
68. https://www.scitepress.org/Papers/2025/134021/134021.pdf
69. https://arxiv.org/abs/2510.23089
70. https://arxiv.org/html/2401.00867v3
71. https://dl.acm.org/doi/10.24963/ijcai.2025/1194
72. https://pennylane.ai/qml/demos/tutorial_tn_circuits/
73. https://inspirehep.net/files/c68cb8407e02410d11b79dfdc859a5c8
74. https://ieeexplore.ieee.org/document/10888238/
75. https://dl.acm.org/doi/10.1145/3547334
76. https://www.youtube.com/watch?v=ryiQLbWVF9w
77. https://www.osti.gov/servlets/purl/2430475
78. https://proceedings.mlr.press/v40/Ge15.pdf
79. https://www.sciencedirect.com/science/article/pii/S0169743924002302
80. https://ietresearch.onlinelibrary.wiley.com/doi/10.1049/qtc2.12086
81. https://ieeexplore.ieee.org/document/10063200/
82. https://arxiv.org/abs/2507.10635
83. https://www.ijcai.org/proceedings/2025/1194.pdf
84. https://thequantumlaend.de/entanglement-i-einsteins-spooky-action-at-a-distance/
85. https://link.aps.org/doi/10.1103/PhysRevA.111.032409
86. https://link.aps.org/doi/10.1103/PhysRevA.72.012318
87. https://ocw.tudelft.nl/wp-content/uploads/sols3_circuits.pdf
88. https://www.ibm.com/quantum/blog/long-range-entanglement
89. https://dl.acm.org/doi/10.1145/3514355
90. https://www.sciencedirect.com/science/article/abs/pii/S0375960113008037
91. https://arxiv.org/abs/2112.11686
92. https://link.aps.org/doi/10.1103/PRXQuantum.3.030312
93. https://quantum-journal.org/papers/q-2020-08-13-307/

94. https://www.nature.com/articles/s41467-019-09039-7
95. https://www.runpod.io/articles/guides/quantum-inspired-ai-algorithms-accelerating-machine-learning
96. https://arxiv.org/pdf/2202.04089.pdf
97. https://dl.acm.org/doi/10.5555/2999611.2999715
98. https://dl.acm.org/doi/10.1145/3520141
99. https://www.jst.go.jp/kisoken/aip/result/event/jst-riken_sympo2022/pdf/5_AIP_Joint_Sympo_Qibin_Zhao.pdf
100. https://www.quandela.com/resources/quantum-computing-glossary/quantum-inspired-algorithms/
101. https://link.aps.org/doi/10.1103/61h8-8qr5
102. https://arxiv.org/abs/1910.10732
103. https://www.jmlr.org/papers/volume22/18-431/18-431.pdf
104. https://arxiv.org/abs/2308.09758
105. https://scipost.org/submissions/scipost_202209_00017v2/
106. https://link.aps.org/doi/10.1103/PhysRevLett.128.020402
107. https://arxiv.org/abs/2412.15826
108. https://www.icfo.eu/download-file/files/event_documents/30032023132546000000.pdf
109. https://ir.soken.ac.jp/record/6647/files/A2402本文.pdf
110. https://inspirehep.net/literature/2834805